Announcements

- If you haven't shown the grader your proof of prerequisite, please do so by 11:59 pm on 01/26/2018 (Friday).

- HW1 is available online already (due on Feb. 14 in class), make sure that you do the right questions. The text of HW1 is also posted online now.

The Grader's Office Hours

- GMCS 425 is grader's office.
- On Monday and Wednesday, the grader will have office hours from 12PM-1:45PM. On Thursday, the grader will have office hours from 2 PM-3:30 PM.

Overview – Canonical Forms

- What are Canonical Forms?
- Minterms
- Index Representation of Minterms
- Sum-of-Minterm (SOM) Representations

Canonical Forms

- **It is useful to specify Boolean functions in a form that:**
  - **Allows comparison for equality.**
  - **Has a correspondence to the truth tables**
- **Canonical Forms in common usage:**
  - **Sum of Minterms (SOM)**

Minterms

- **<u>Minterms</u> are AND terms with every variable present in either true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n minterms for *n* variables.**
- **<u>Example:</u> Two variables (X and Y)produce 2 x 2 = 4 combinations:**

  XY **(both normal)**

  $X\overline{Y}$ **(X normal, Y complemented)**

  $\overline{X}Y$ **(X complemented, Y normal)**

  $\overline{X}\,\overline{Y}$ **(both complemented)**

- **Thus there are <u>four minterms</u> of two variables.**

Minterms

- **Examples: Two variable minterms and maxterms.**

Index	Minterm	Maxterm
0	$\overline{x}\,\overline{y}$	$x + y$
1	$\overline{x}\,y$	$x + \overline{y}$
2	$x\,\overline{y}$	$\overline{x} + y$
3	$x\,y$	$\overline{x} + \overline{y}$

- **The index above is important for describing which variables in the terms are true and which are complemented.**

Standard Order

- **Minterms are designated with a subscript**
- **The subscript is a number, corresponding to a binary pattern**
- **The bits in the pattern represent the complemented or normal state of each variable listed in a standard order.**
- **All variables will be present in a minterm and will be listed in the <u>same order</u> (usually alphabetically)**
- **Example: For variables a, b, c:**
  - **Maxterms:** $(a + b + \bar{c})$, $(a + b + c)$
  - **Terms:** $(b + a + c)$, $a\,\bar{c}\,b$, **and** $(c + b + a)$ **are NOT in standard order.**
  - **Minterms:** $a\,\bar{b}\,c$, $a\,b\,c$, $\bar{a}\,\bar{b}\,c$
  - **Terms:** $(a + c)$, $\bar{b}\,c$, **and** $(\bar{a} + b)$ **do not contain all variables**

Purpose of the Index

- **The index for the minterm or maxterm, expressed as a binary number, is used to determine whether the variable is shown in the true form or complemented form.**
- **For Minterms:**
  - **"1" means the variable is "Not Complemented" and**
  - **"0" means the variable is "Complemented".**
- **For Maxterms:**
  - **"0" means the variable is "Not Complemented" and**
  - **"1" means the variable is "Complemented".**

Index Example in Three Variables

- **Example: (for three variables)**
- **Assume the variables are called X, Y, and Z.**
- **The standard order is X, then Y, then Z.**
- **The <u>Index 0</u> (base 10) = 000 (base 2) for three variables). All three variables are complemented for <u>minterm 0</u> ($\overline{X}, \overline{Y}, \overline{Z}$) and no variables are complemented for <u>Maxterm 0</u> (X,Y,Z).**
  - **Minterm 0, called m_0 is $\overline{X}\,\overline{Y}\,\overline{Z}$.**
  - **Maxterm 0, called M_0 is (X + Y + Z).**
  - **Minterm 6 ?**

Index Examples – Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}$	$a + b + c + d$
1	0001	$\overline{a}\,\overline{b}\,\overline{c}\,d$	?
3	0011	?	$a + b + \overline{c} + \overline{d}$
5	0101	$\overline{a}\,b\,\overline{c}\,d$	$a + \overline{b} + c + \overline{d}$
7	0111	?	$a + \overline{b} + \overline{c} + \overline{d}$
10	1010	$a\,\overline{b}\,c\,\overline{d}$	$\overline{a} + b + \overline{c} + d$
13	1101	$a\,b\,\overline{c}\,d$	?
15	1111	$a\,b\,c\,d$	$\overline{a} + \overline{b} + \overline{c} + \overline{d}$

Function Tables for Both

- Minterms of 2 variables

x y	m_0	m_1	m_2	m_3
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Maxterms of 2 variables

x y	M_0	M_1	M_2	M_3
0 0	0	1	1	1
0 1	1	0	1	1
1 0	1	1	0	1
1 1	1	1	1	0

Observations

- **In the function tables:**
  - **Each minterm has one and only one 1 present in the 2^n terms (a minimum of 1s). All other entries are 0.**
  - **Each maxterm has one and only one 0 present in the 2^n terms All other entries are 1 (a maximum of 1s).**
- **We can implement any function by "ORing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.**
- **We can implement any function by "ANDing" the maxterms corresponding to "0" entries in the function table. These are called the maxterms of the function.**
- **This gives us two canonical forms:**
  - **Sum of Minterms (SOM)**
  - **Product of Maxterms (POM)**

for stating any Boolean function.

Minterm Function Example

- Example: Find $F_1 = m_1 + m_4 + m_7$
- $F1 = \overline{x}\ \overline{y}\ z + x\ \overline{y}\ \overline{z} + x\ y\ z$

x y z	index	m_1	+	m_4	+	m_7	$= F_1$
0 0 0	0	0	+	0	+	0	= 0
0 0 1	1	1	+	0	+	0	= 1
0 1 0	2	0	+	0	+	0	= 0
0 1 1	3	0	+	0	+	0	= 0
1 0 0	4	0	+	1	+	0	= 1
1 0 1	5	0	+	0	+	0	= 0
1 1 0	6	0	+	0	+	0	= 0
1 1 1	7	0	+	0	+	1	= 1

Minterm Function Example

- **F(A, B, C, D, E) = $m_2 + m_9 + m_{17} + m_{23}$**
- **F(A, B, C, D, E) =**

Canonical Sum of Minterms

- **Any Boolean function can be expressed as a <u>Sum of Minterms</u>.**
  - **For the function table, the <u>minterms</u> used are the terms corresponding to the 1's**
  - **For expressions, <u>expand</u> all terms first to explicitly list all minterms. Do this by "ANDing" any term missing a variable v with a term $(v + \overline{v})$.**
- **Example: Implement $f = x + \overline{x}\ \overline{y}$ as a sum of minterms.**

First expand terms: $f = x(y + \overline{y}) + \overline{x}\ \overline{y}$

Then distribute terms: $f = xy + x\overline{y} + \overline{x}\ \overline{y}$

Express as sum of minterms: $f = m_3 + m_2 + m_0$

Another SOM Example

- Example: $F = A + \overline{B}\,C$
- There are three variables, A, B, and C which we take to be the standard order.
- Expanding the terms with missing variables:

- Collect terms (removing all but one of duplicate terms):
- Express as SOM:

Shorthand SOM Form

- **From the previous example, we started with:**

$$F = A + \overline{B}\,C$$

- **We ended up with:**

$$F = m_1+m_4+m_5+m_6+m_7$$

- **This can be denoted in the formal shorthand:**

$$F(A,B,C) = \Sigma_m(1,4,5,6,7)$$

- **Note that we explicitly show the standard variables in order and drop the "m" designators.**

Standard Forms

- **<u>Standard Sum-of-Products (SOP) form:</u> equations are written as an OR of AND terms**
- **<u>Standard Product-of-Sums (POS) form:</u> equations are written as an AND of OR terms**
- **Examples:**
  - **SOP:** $A\,B\,C + \overline{A}\,\overline{B}\,C + B$
  - **POS:** $(A + B) \cdot (A + \overline{B} + \overline{C}) \cdot C$
- **These "mixed" forms are <u>neither SOP nor POS</u>**
  - $(A\,B + C)\,(A + C)$
  - $A\,B\,\overline{C} + A\,C\,(A + B)$

Standard Sum-of-Products (SOP)

- **A sum of minterms form for *n* variables can be written down directly from a truth table.**
  - **Implementation of this form is a two-level network of gates such that:**
  - **The first level consists of *n*-input AND gates, and**
  - **The second level is a single OR gate (with fewer than 2^n inputs).**
- **This form often can be simplified so that the corresponding circuit is simpler.**

Standard Sum-of-Products (SOP)

- **A Simplification Example:**
- $\mathbf{F(A, B, C) = \Sigma m(1,4,5,6,7)}$
- **Writing the minterm expression:**

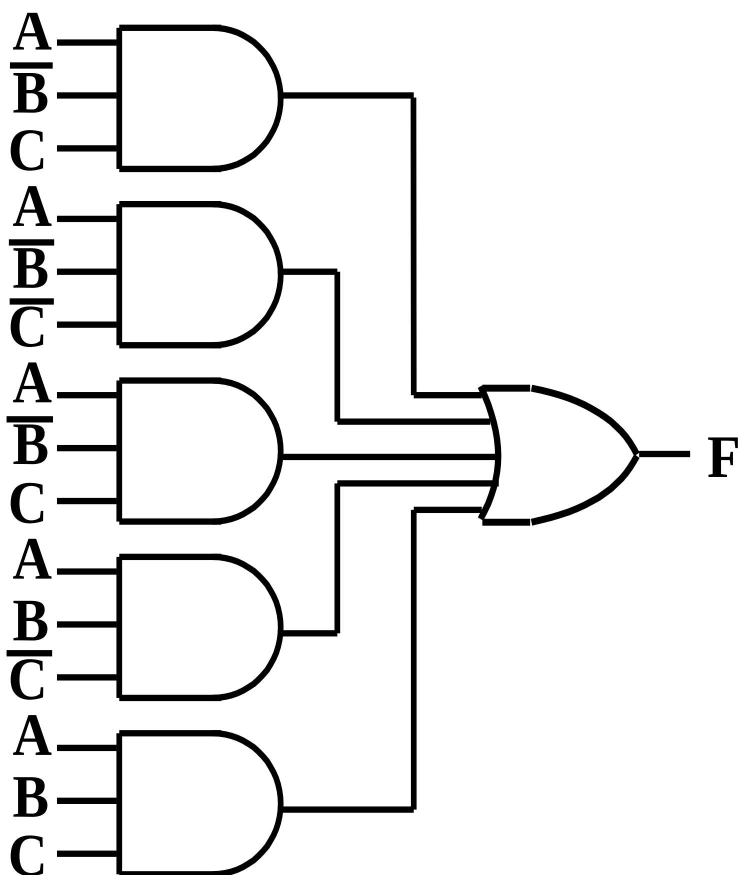
  $\mathbf{F = \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + AB\overline{C} + ABC}$
- **Simplifying:**

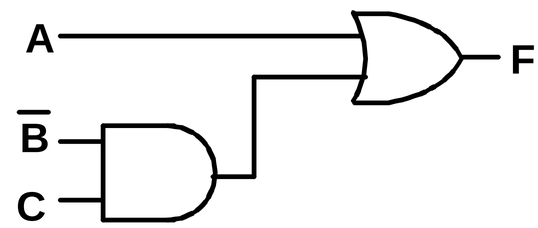
  F =

- **Simplified F contains 3 literals compared to 15 in minterm F**

AND/OR Two-level Implementation of SOP Expression

- **The two implementations for F are shown below – it is quite apparent which is simpler!**





Weekly Exercise 1-2

- Problems (P102 on text book)
- 2-1; 2-2; 2-7; 2-9(c); 2-10(c); 2-12(b)
- Get familiar with Table 2-6 (P49 on text book)
- Get familiar with the 6 identities on P53.

Hi-Impedance Outputs

- Logic gates introduced thus far
  - have 1 and 0 output values,
  - cannot have their outputs connected together, and
  - transmit signals on connections in only one direction.
- Three-state logic adds a third logic value, Hi-Impedance (Hi-Z), giving three states: 0, 1, and Hi-Z on the outputs.
- The presence of a Hi-Z state makes a gate output as described above behave quite differently:
  - “1 and 0” become “1, 0, and Hi-Z”
  - “cannot” becomes “can,” and
  - “only one” becomes “two”

Hi-Impedance Outputs **(continued)**

- What is a Hi-Z value?
  - The Hi-Z value behaves as an open circuit
  - This means that, looking back into the circuit, the output appears to be disconnected.
  - It is as if a switch between the internal circuitry and the output has been opened.
- Hi-Z may appear on the output of any gate, but we restrict gates to:
  - a 3-state buffer, or
  - a transmission gate,

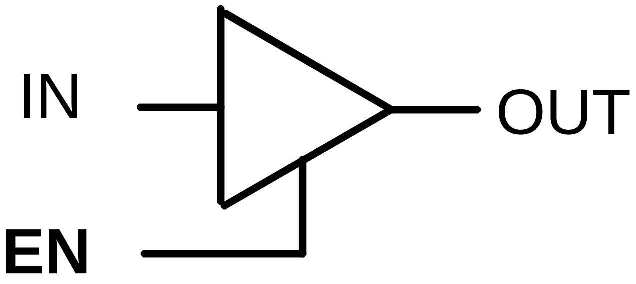
each of which has one data input and one control input.

The 3-State Buffer

- For the symbol and truth table, IN is the data input, and EN, the control input.
- For EN = 0, regardless of the value on IN (denoted by X), the output value is Hi-Z.
- For EN = 1, the output value follows the input value.
- Variations:
  - Data input, IN, can be inverted
  - Control input, EN, can be inverted

  by addition of "bubbles" to signals.

Symbol



Truth Table

EN	IN	OUT
0	X	Hi-Z
1	0	0
1	1	1

Terms of Use

-
- The following terms of use apply in addition to the standard Pearson Education Legal Notice.
- Permission is given to incorporate these materials into classroom presentations and handouts only to instructors adopting Logic and Computer Design Fundamentals as the course text.
- Permission is granted to the instructors adopting the book to post these materials on a protected website or protected ftp site in original or modified form. All other website or ftp postings, including those offering the materials for a fee, are prohibited.
- You may not remove or in any way alter this Terms of Use notice or any trademark, copyright, or other proprietary notice, including the copyright watermark on each slide.
- Return to Title Page